Financial Economics
Problem Set 1

LEC, SJTU

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1. Let $p \in \mathbb{R}^\ell$ be a price vector, let $\omega \in \mathbb{R}^\ell$ be an agent’ s endowment, and let x and x' be two distinct consumption bundles. Prove the following:

(a) If both x and x' satisfy the budget constraint with equality, then any linear combination $z := \lambda x + (1-\lambda)x$, with $\lambda \in \mathbb{R}$, also satisfies the budget constraint with equality.

(b) If both x and x' satisfy the weak budget constraint, then any convex combination $z := \lambda x + (1-\lambda)x$, with $\lambda \in [0,1]$, also saisfies the weak budget constraint.

2. Consider an agent who lives for two periods and who faces the decision problem of how much to consume now and how much to save. w is his first period income, his second period income is zero (he’ s retired then), and there is a guaranteed gross real interest rate ρ on any savings:

(a) Suppose the agent has an additively separable utility function of the following form:

$$u(x_1, x_2) = \ln x_1 + \delta \ln x_2$$

We use the logarithm as the period utility function, indicating the amount of utils you get from consuming some quantity in one period. δ is a weight which measures your relative valuation of consuming now versus consuming later. It measures therefore your time preference and is usually called the discount factor. We typically assume that $0 < \delta < 1$, meaning that people are not patient and would rather enjoy consumption now than later. With these preferences, compute his optimal saving, as a function of ρ and δ?

(b) Now consider the preferences represented by this utility function,

$$\tilde{u}(x_1, x_2) = x_1^\gamma x_2^\varepsilon.$$

Do you have any prior about the solution? If yes, write it down. If no, or if you are unsure, compute the optimal saving, as a function of ρ, γ , and ε. Compare your result with your answer to the previous problem and comment.

3. (a) Consider a risk-free bond paying today’ s purchasing power of \$100 a year from now. This means that the bond will pay tomorrow an amount of money that suffices to buy a bundle of commodities that cost \$100 today, so this is a real or inflation indexed bond. Suppose this bond costs \$97.73. What is the real interest rate?

(b) Suppose there is a second bond, paying two years from now the purchasing power that \$100 has today. Suppose this bond costs \$95.02 today. On the basis of these figures, is the real interest rate next year lower or higher than this year?

4. Suppose there are two states and three financial assets, a risk-free bond with state-contingent cash flows (100, 100), a risky bond that pays only in state 2, with cash flow (0, 100), and a share with cash flow (20, 35). Can you find a portfolio containing only shares and risky bonds that reproduces 40 risk-free bonds?

5. Consider a situation in which there are five states, and suppose you can observe the prices of the Arrow securities. They are (0.1225, 0.2451, 0.3676, 0.0613, 0.1838).

(a) Compute the price of a risk-free bond and the risk-free rate of return.

(b) What are the risk-neutral probabilities of the five states?

(c) How much does a hypothetical asset cost with cash flow (5, 5, 2, 7, 4)?

6. There are two dates: At date 1 there are three states; at date 0 there is trade in assets. There are two basic assets whose return vectors in current dollars are $r_1 = (64, 16, 4)$ and $r_2 = (0, 0, 1)$ The market prices of these assets are $q_1 = 32$ and $q_2 = 1$, respectively. In the following you are asked to price by arbitrage a variety of derived assets:

(a) Suppose that one unit of a derived asset is described as "One unit of this asset confers the right to buy one unit of asset 1 at 75% of its spot value in period 1 (after the state of the world occurs)". Write the return vector of this asset and price it.

(b) The situation is the same as in (a) except that the asset is modified to read "One unit of this asset confers the right to buy one unit of asset 1 at 75% of its spot value in period 1 (after the state of the world occurs) provided the spot value is at least 10".

(c) Suppose that the asset is as in (b) except that "at least 10" is replaced by "at least 19". Write the return vector and argue that this asset cannot be priced by arbitrage with the available primary assets.

(d) How would the analysis in (c) differ if we had in addition a riskless asset with a price equal to 1?

7. Consider a binary risk: you will either win H with probability π (e.g., the jackpot of the national lottery), or lose L (the price of participating in the lottery) with probability $1`\pi$. You can expose yourself in a continuous manner to this risk, meaning that you can buy x tickets, where x is any real number. Assume that you have preferences that can be represented with a risk-averse von Neumann–Morgenstern utility function.

(a) Prove that you will take some of this risk ($x > 0$) if the expected payoff, $\pi H`(1`\pi)L$, is positive.

(b) The expected payoff of the national lottery is negative. Why do some people participate? Have you personally participated in the past?

8. Between 1930 and 1999, real GDP per capita in the USA grew 2.24% per year on average, with a standard deviation of 5.21%. Suppose the coefficient of relative risk aversion of the representative agent (the one constructed with the competitive SWF) is 2.

(a) What is the certainty equivalent growth rate?

(b) How do you think the result would change qualitatively if we considered real consumption instead of real GDP?

(c) Given this, do you think that business cycles are a major economic problem for society?

(d) Try to think of arguments why something might be wrong with this analysis.